Hydrodynamic Free Convection Flow of A Rotating Visco-elastic Fluid Past An Isothermal Vertical Porous Plate With Mass Transfer

S. Biswal, G.S. Ray, A. Mishra

Abstract - The effect of free convection and mass transfer in the unsteady flow of an incompressible electrically conducting visco-elastic past an isothermal vertical porous plate with constant suction normal to the plate has been studied. The effects of permeability parameter (Kp) of the porous medium, rotation parameter (R), Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm), frequency parameter (ω) and the heat source parameter (α0) on the transient primary and secondary velocity field, temperature field and the rate of heat transfer have been investigated with the help of graphs and tables.

Key words - Hydrodynamic flow, mass transfer, rotating fluid, porous medium, isothermal plate.

1. INTRODUCTION

THE problem of hydrodynamic free convection flow of a rotating viscoelastic fluid has received a considerable attention of many researchers because of its applications in cosmical and geophysical science. These problems are of general interest in the field of atmospheric and oceanic circulations, nuclear reactors, power transformers and in the field of scientific and industrial research. Permeable porous plates are used in the filtration process and also for a heated body to keep its temperature constant and to make the heat insulation of the surface more effective.

Several authors have discussed the flow of a viscous fluid in a rotating system in the presence and absence of magnetic field. Reptis and Singh [1] have reported the effect of rotation on the free convection MHD flow past an accelerated vertical plate. Singh[2] has studied the unsteady free convection flow of a viscous liquid through a rotating porous medium. Dash and Biswal [3] have investigated the effect of at infinite vertical porous plate with time dependent temperature and concentration.

Rath and Bastia[7] have analysed the steady flow and heat transfer in a visco-elastic fluid between two coaxial rotating disks. Mukherjee and Mukherijee[8] have studied the unsteady axisymmetric rotational flow of elastico-viscous liquid. Datta and Jana[9] have investigated the problem of flow and heat transfer in an elastico-viscous liquid over an oscillating plate in a rotating frame.

The present study considers the simultaneous effect of heat and mass transfer on the hydrodynamic free convection flow of a rotating viscoelastic fluid past an infinite vertical isothermal porous plate.

2. Formulation of the problem

Let us consider the flow of a rotating viscoelastic incompressible fluid σ in a medium past an infinite vertical isothermal porous plate. Let x and y-axis be in the plane of the plate and z-axis normal to the plate and let u, v, w be the velocity components of the fluid in *x, y* and z direction respectively. Both the liquid and the plate are considered in a state of rigid body rotation about z-axis with uniform angular velocity Ω. Further, Let us assume that the constant heat source Q (absorption type $Q = -Q_0 (T-T_\infty)$ is at $z = 0$ and the suction velocity at the plate $w = -w_0$ where w_0 is a positive real number. Here we have neglected buoyancy effect. Since the plate is infinite in extent, all physical variables depend on z and t only. Considering $u + iv = q$ and using non-dimensional quantities the equations governing the flow (dropping the asterisks) are

$$\frac{\partial q}{\partial t}-\frac{\partial q}{\partial z}+\left(2iR+\frac{1}{K_p}\right)(q-U)=\frac{\partial U}{\partial t}+\frac{\partial^2 q}{\partial z^2}+R_c\frac{\partial^3 q}{\partial z^2\partial t}+G_rT+G_mC$$

---------------------(1)

$$\frac{\partial^2 T}{\partial z^2}+\Pr\frac{\partial T}{\partial z}-\Pr\frac{\partial T}{\partial t}-\alpha_0 T=0$$

---------------------(2)

$$\frac{\partial^2 C}{\partial z^2}+Sc\frac{\partial C}{\partial z}-Sc\frac{\partial C}{\partial t}=0$$

--------------------(3)

The non-dimensional boundary conditions are

$q = 1 + \varepsilon e^{i\omega t}$, $(T = 1 + \varepsilon e^{i\omega t}, \omega<0)$, $C = 1 + \varepsilon e^{i\omega t}$ at $z = 0$

$q = (1 + \varepsilon e^{i\omega t}, \omega>0)$, $T \to 0$, $C = 0$ at $z \to\infty$ --------------------- (4)

The non-dimensional quantities introduced in equations (1) – (3) are defined as

$$z^* = w_0\frac{z}{v},\ t^* = w_0\frac{t}{v},\ u^* = \frac{u}{U_0},$$

$$K_p^* = K_p' \, w_0^2 / v^2$$

Where K_p' is the dimensional permeability parameter and K_p^* is the non-dimensional permeability parameter.

$$q = \frac{u}{U_0}+i\frac{v}{U_0},\ T=\frac{T-T_\infty}{T_w-T_\infty},\ P_r=\frac{v\rho C_p}{K},$$

$$R=\frac{\Omega v}{W_0^2},\ R_c=\frac{K_0\omega_0^2}{v^2}$$

$$G_m = \frac{\rho g\beta^*(C_w-C_\infty)}{U_0w_0^2},\ G_r = \frac{vg\beta_0}{U_0w_0^2}(T_w-T_\infty),$$

$$V = v' / U_0$$

Where P_r : Prandtl number

G_r : Grashof number,

M : Magnetic parameter,

R : Rotation parameter

K_p : Permeability parameter,

α_0 : heat source parameter,

U : free steam velocity

R_c : non-Newtonian parameter

3. **Solution of the equations**:

In order to solve the equation [1] – [3] we assume velocity, temperature and concentration of the liquid in the neighborhood of the plate as

$Q = (1 -q_0) + \varepsilon e^{i\omega t} (1 -q_1)$ ---------------------(5)

$T = T_0 + \varepsilon T_1' e^{i\omega t}$ --------------------- (6)

$C = C_0 + \varepsilon e^{i\omega t} C_1$

--------------------- (7)

Where

$(1 - q_1)\, e^{i\omega t} = (M_r + I\, M_i)\,(\cos \omega t + i \sin \omega t)$

$T_1\, e^{i\omega t} = (T_r + iT_i)\,(\cos \omega t + i \sin \omega t)$

$C_1\, e^{i\omega t} = (C_r + iC_i)\,(\cos \omega t + i \sin \omega t)$

For free stream (when $\varepsilon \ll 1$)

$U(t) = 1 + \varepsilon\, e^{i\omega t}$ ------------------------(8)

The transient primary velocity and temperature profiles can be deduced from equations (5) and (6) for $\omega t = \pi/2$

Hence $u(z, t) = u_0(z) - \varepsilon M_i$

--------------------- (9)

$T = T_0 - \varepsilon T_1$

--------------------- (10)

Where $u_0(z) + iv_0(z) = 1 - q_0$

Using equations (5) – (8) in equations (1), (2) and (3) and equating harmonic and non-harmonic terms, we get

$(1+i\omega R_c)$

$$q_1'' - Vq_1' - \left[\frac{1}{K_p} + (2iR + i\omega)\right] q_1 = -G_r T_1 - G_m C_1$$

--------------------- (11)

$$q_1'' - Vq_0' - \left(\frac{1}{K_p} + 2iR\right) q_0 = -G_r T_0 - G_m C_0$$

--------------------- (12)

$$T_0'' - P_r V T_0' - \alpha_0 T_0 = 0$$

--------------------- (13)

$$T_1'' - P_r V T_1' - (i\omega P_r + \alpha_0) T_1 = 0$$

--------------------- (14)

$$C_0'' + S_c C_1' = 0$$

--------------------- (15)

$$C_1'' + S_c C_1' - i\omega S_c C_1 = 0$$

--------------------- (16)

The boundary conditions are

$q_0 = 0,\ T_0 = 1,\ C_0 = 1,\ q_1 = 0,\ T_1 = 1,\ C_1 = 0$ at $z = 0$

$q_0 \to 0,\ T_0 \to 0,\ C_0 \to 0,\ q_1 \to 0,\ T_1 \to 0,\ C_1 \to 0$ at $z \to \infty$ --------------------- (17)

Solving equations (11) – (16) with boundary conditions (17) we obtain for $R_c \ll 1$ and ω small,

$C_0 = e^{-S_c z}$

$C_1 = e^{D_1 z}$

$T_0 = e^{D_2 z}$

$T_1 = e^{D_3 z}$

$q_0 =$

$$\frac{Gr\left(e^{D_2 z} - e^{D_2 z}\right)}{(D_2 - D_4)(D_2 - D_5)} + \frac{Gm\left(e^{-scz} - e^{D_3 z}\right)}{(Sc + D_4)(Sc + D_5)}$$

$q_1 =$

$$\frac{Gr\left(e^{D_3 z} - e^{D_7 z}\right)}{(D_3 - D_6)(D_3 - D_7)} + \frac{Gm\left(e^{D_3 z} - e^{D_7 z}\right)}{(D_1 - D_6)(D_1 - D_7)}$$

Putting values of T_0, T_1 in equation (6)

C_0, C_1 in equation (7) and q_0, q_1 in equation (5) we get

$T = e^{D_2 z} + \in e^{i\omega t}\left(e^{D_3 z}\right)$

$C = e^{-S_c z} + \in e^{i\omega t}\left(e^{D_1 z}\right)$

$$q = 1 - \frac{Gr\left(e^{D_2 z} - e^{D_2 z}\right)}{(D_2 - D_4)(D_2 - D_5)} + \frac{Gm\left(e^{-scz} - e^{D_3 z}\right)}{(S_c + D_4)(S_c + D_5)}$$

$+\varepsilon e^{i\omega t}$

$$\left[1 - \frac{Gr\left(e^{D_3 z} - e^{D_7 z}\right)}{(D_3 - D_6)(D_3 - D_7)} - \frac{Gm\left(e^{D_3 z} - e^{D_7 z}\right)}{(D_1 - D_6)(D_1 - D_7)}\right]$$

Separating real and imaginary parts

$u = 1 - G_r (B_4 B_{12} - B_5 B_{13}) - G_m (B_6 B_{14}$

$- B_7 B_{13}) + \varepsilon H_1 \cos \omega t - \varepsilon H_2 \sin \omega t$

$v = \varepsilon H_2 \cos \omega t + \varepsilon H_1 \sin \omega t - G_r (B_5 B_{12} + B_4 B_{13}) - G_m (B_7 B_{14} + B_6 B_{13})$

Where

$H_1 = 1 - (B_8 B_{16} - B_9 B_{17}) G_r - (B_{10} B_{18} - B_3 B_{11}) G_m,$

$H_2 = - G_r (B_0 B_{16} + B_8 B_{17}) - G_m (B_{11} B_{18} + B_3 B_{10}),$

$$D_1 = \sqrt{\left(1 + \frac{4}{K_p}\right)^2 + 64R^2}$$

$$D_2 = \sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + 16\omega^2 P_r^2},$$

$$D_3 = \sqrt{\left(1 + 4M^2 \frac{4}{K_p}\right)^2 + \left(8R + 4\omega\right)^2},$$

Taking the value of V=1.

The values of the other constants involved are omitted here to save space.

4. Results and Discussions

The problem of hydrodynamic free convection and mass transfer flow of a rotating viscous fluid past an isothermal vertical porous plate has been considered. The effects of porosity parameter (K_p), rotation parameter (R), Grashof numbers (G_r, G_m), frequency parameter (ω) and the heat source parameter (α_0) on the transient primary velocity, secondary velocity, temperature distribution profiles and the rate of heat transfer have been discussed to observe the physical significance and the mystery of the problem. To be realistic Prandtl number is chosen as 0.71 (the water vapour) and 7.0 (for water) approximately at 1 atmosphere and 25^0 C). From physical point of view G_r<0 corresponds to cooling of the plate and G_r<0 corresponds to heating of the plate by free convection currents.

The transient primary velocity profiles are shown in Fig. 1. It is observed that the transient velocity u increases regressively and after attaining the maximum value decreases asymptotically and ultimately attains steady state. Comparing the curves (4 and 5) of figures (1), it is observed that the porosity parameter (K_p) increases the primary velocity. The rotation parameter has negligible effect on the primary velocity while it decreases the secondary velocity appreciably (curves 4 and 6). Comparing the curves (4, 8) and (2, 3), it is evident that the Grashof number for heat transfer (G_r) and the Grashof number for mass transfer (G_m) have accelerating effect on both the components of the velocity field. On careful observation of curves (4, 9) and (2, 4), it is observed that Prandtl number (P_r) and heat source parameter (α_0) have decelerating effect on the transient velocity. The frequency parameter has negligible effect on the velocity field.

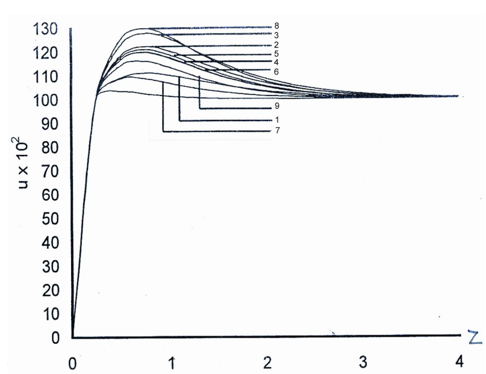
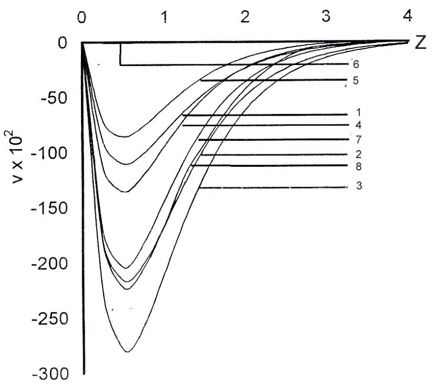


Fig 1 : Effect of different parameters on transient primary velocity profile when ε= 0.02



Curves	K_p	R	G_r	α_0	ω	P_r	G_m
1	1	.2	2	1	5	.71	0
2	1	.2	2	1	5	.71	2
3	1	.2	2	1	5	.71	3
4	1	.2	2	1	5	1	2
5	2	.2	2	1	5	1	2
6	1	.4	2	1	5	1	2
7	1	.2	2	1	5	1	2
8	1	.2	4	1	5	1	2
9	1	.2	4	1	10	1	2

Fig 2: Effect of different parameters on transient secondary velocity profile when ε=0.02

The effects of different parameter on the transient secondary velocity have been shown in Fig.2. It is marked that the velocity V decreases with modified Grashof number G_m (curves 1, 2 and 3). This is valid for water vapour or air. But, the velocity 'V' rises for liquid sodium (P_r=1). As the value of rotation parameter R increase, the transient secondary velocity also rises (curve 5). The variation of frequency parameter ω does not produce any appreciable variation in the velocity 'V'. An increase in Grashof number G_r reduces the velocity 'V'. For water (P_r = 7.0), the velocity 'V' falls in comparison to that of liquid sodium (P_r = 1.0)

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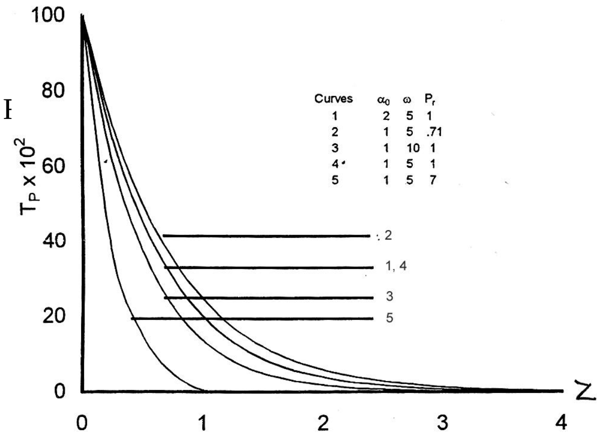

of the tempe rature field. An increa se in frequency parameter leads to an oscillatory ... dary temperature ... d for the transient ... ves 3 and 4). The ... ates the transient ... oes not affect the

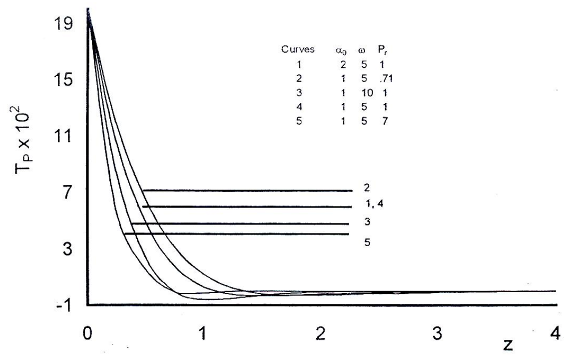

Fig 3 : Effect of different parameters on transient primary temperature profiles when ε= 0.02, ωt = $\pi/2$

Table (1 and 2) show the effects of α_0, P_r and ω on the transient primary rate of heat transfer (Nu_p) and transient secondary rate of heat ... 1 and ωt = ... reveal that ... eter (α_0>0), α_0 increase Nu_p and Nu_s and reverse effect is

Curves	K_p	R	G_r	α_0	ω	P_r	G_m
1	1	.2	2	1	5	.71	0
2	1	.2	2	1	5	.71	2
3	1	.2	2	1	5	.71	3
4	1	.2	2	1	5	1	2
5	1	.4	2	1	5	1	2
6	1	.2	2	1	5	1	2
7	1	.2	4	1	5	1	2
8	1	.2	2	1	5	7	2

noted for generation type that source parameter (α_0>0). It is interesting to note that an increase in frequency parameter leads to an oscillatory effect on both components of transient heat transfer.

Fig. 4 : Effect of different parameters on transient secondary temperature profiles when ε = 0.02, ωt = $\pi/2$

Table: 1

Variation of transient primary (Nu_p) and secondary (Nu_s) heat transfer at ε = 0.01 and ωt = $\pi/2$ (absorption type)

α_0	P_r	ω	Nu_p	Nu_s
1.0	0.71	5.0	– 0.011391	0.019139
2.0	0.71	5.0	– 0.010032	0.021261
1.0	7.00	5.0	– 0.034770	0.085340
2.0	7.00	5.0	– 0.034309	0.086018
1.0	0.71	10.0	– 0.017410	0.023950
2.0	0.71	10.0	– 0.016239	0.025409

Table: 2

Variation of transient primary (Nu_p) and secondary (Nu_s) heat transfer at ε = 0.01 and ωt = $\pi/2$ (generation type)

α_0	P_r	ω	Nu_p	Nu_s
– 1.0	0.71	5.0	– 0.015049	0.015359
– 2.0	0.71	5.0	– 0.017161	0.013911
– 1.0	7.00	5.0	– 0.035720	0.084011
– 2.0	7.00	5.0	– 0.036209	0.083340
– 1.0	0.71	10.0	– 0.020042	0.021279
– 2.0	0.71	10.0	– 0.021469	0.020091

5. Conclusion

The following conclusions are drawn for the transient primary and secondary velocity field, temperature field and the rate of heat transfer.

1. Heat source parameter (α_0) and Prandtl number (P_r) have decelerating effect on both components of the transient velocity field. The effect of the heat source parameter (α_0) is more pronounced than the Prandtl number (P_r). The Prandtl number also increases the temperature field.
2. The Grashof number for heat transfer

(G_r) and Grash of number for mass transfer (G_m) enhance both components of the velocity of the flow field.

3. The porosity parameter (K_p) accelerates the transient primary velocity field while it does not affect the transient secondary velocity field.
4. Both components of the velocity field decrease with the increase in rotation parameter but the effect is more pronounced in the case of the secondary velocity. The effect of frequency parameter(ω) is significant for both the components of temperature field and the rate of heat transfer and it has a negligible effect on the velocity field.
5. The heat source parameter enhances both components of rate of heat transfer for heat sink and the reverse effect is observed for heat source.

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